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$$= \frac{-x^{n+1}[n(x-1)-1]^2 + x(x+1)-x^{n+2}}{(1-x)^3}.$$

Putting  $-x$  in place of  $x$ , we have

$$\begin{aligned} S &= ax - bx^2 + cx^3 - dx^4 + \dots \\ &= a \left[ \frac{x}{1+x} \right] - 4a \left[ \frac{x}{1+x} \right]^2 + 4^2 a \left[ \frac{x}{1+x} \right]^3 - \dots \end{aligned}$$

Putting  $x = 1$ , we have

$$a - b + c - d + e - f + \dots \text{ ad inf. } = \frac{1}{2}a - \frac{1}{4}4a + \frac{1}{8}4^2a - \frac{1}{16}4^3a + \dots$$

Thus we find,  $1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \dots \text{ ad inf. } = \frac{1}{2},$

$$1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \dots \text{ ad inf. } = \frac{1}{4},$$

$$1 - 3 + 5 - 7 + 9 - 11 + 13 - \dots \text{ ad inf. } = 0,$$

$$1 - 4 + 9 - 16 + 25 - 36 + 49 - \dots \text{ ad inf. } = 0.$$

The formula  $ax - bx^2 + cx^3 - dx^4 + \dots$

$$= a \left[ \frac{x}{1+x} \right] - 4a \left[ \frac{x}{1+x} \right]^2 + 4^2 a \left[ \frac{x}{1+x} \right]^3 - \dots$$

is also very useful, sometimes, to change a series into one that converges more rapidly. For instance,

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

$$= \frac{x}{1+x} \left\{ 1 + \frac{1}{2} \left[ \frac{x}{1+x} \right] + \frac{1}{3} \left[ \frac{x}{1+x} \right]^2 + \frac{1}{4} \left[ \frac{x}{1+x} \right]^3 + \dots \right\}.$$

$$\tan^{-1}x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots = \frac{1}{x} (x^2 - \frac{1}{3}x^4 + \frac{1}{5}x^6 - \frac{1}{7}x^8 + \dots)$$

$$= \frac{x}{1+x^2} \left\{ 1 + \frac{2}{3} \left[ \frac{x^2}{1+x^2} \right] + \frac{2 \cdot 4}{3 \cdot 5} \left[ \frac{x^2}{1+x^2} \right]^2 + \frac{2 \cdot 4 \cdot 6}{3 \cdot 5 \cdot 7} \&c. \right\}.$$

The above method of the summation of series is by *Euler*. He published it in his *Inst. Calc. Diff.*

QUERY BY E. HANCOCK, ELGIN, ILL. — If a wheel of 60 teeth work in a pinion of 10 leaves, the distance between their centres being .455 in., the diameters of the pitch circles will be .78 in., =  $a$ , and .13 in., =  $b$ , respectively. Allowing one-half for space between the teeth, each tooth will occupy a central angle of  $3^\circ$ , =  $c$ . Suppose the teeth to terminate at the intersection of two similar epicycloidal curves the diameter of whose generating circle is .065, =  $\frac{1}{2}b$ ; what must be the *addendum* to the wheel outside of the pitch circle, i. e., what is the length of the teeth, outside of the pitch circle?